Fault Tolerant Control of Quadrotor
:via Feedback Linearization & Sliding Mode & Backstepping

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***Abstract*—In this paper, the feedback linearization, sliding mode and backstepping control methods are used to control the nonlinear quadrotor vehicle in fault-free condition and faulty condition. First, derived a mathematical model of the quadrotor. Then applied three different nonlinear control methods and showed the convergence stability and controllable variables. Moreover one of the blade goes faulty condition, we derived how the controller have to be changed. We showed controllable and stability in fault tolerant control using the controllers previously constructed by changing the input variables.**

I. INTRODUCTION

Quadrotor is one of the general flight system used for multiple tasks. But it is complex to control due to the versatility and maneuverability. It has been researched to control the quadrotors with various methods[1]. To control the non-linear dynamics of quadrotor in fault-free condition, feedback linearization by Voos(2009)[2], sliding mode by Xu(2006)[3], and backstepping control by Bouabdallah(2005)[4] can be used. Despite the control system is stabilized, it is impossible to ensure the system would always operate properly. So the fault tolerant control is needed for the malfunction of the actuators or sensors. For each methods to control the quadrotor under fault-free condition can also used in faulty condition. Using feedback linearization by Freddi(2011)[5], sliding mode by Sharifi(2010)[6] and backstepping control by Zhang(2010)[7]. In this project, we will design the controller with three methods previously mentioned. Then redesign the controller under fault condition on one actuator.

A. Objectives

In any dynamic systems, the basic step will be modeling the system. So, we will understand the steps to express the dynamics of quadrotor and get a model of it. We can control the four independent actuators in blades, so with the model we obtained, we can show the stability about four state variables altitude, roll, pitch and yaw angle. There are various methods to control the system, but in this project, we will apply Feedback Linearization, Sliding Mode and Backtepping controls. Not only that, we will check how the controller changed in faulty condition. The assumption would be the one actuator is in faulty, so the quadrotor will loose the stability of one variable. So we will understand how to maintain the stability by loosing the control of yaw, then redesign the FTC controller to stabilize roll, pitch, and altitude.

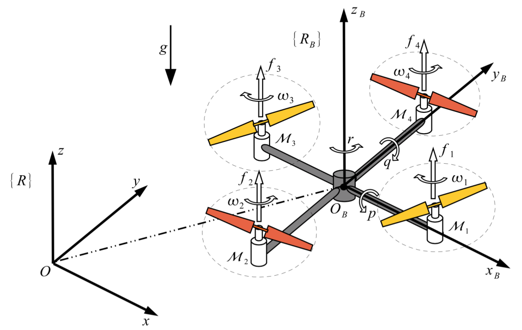

Figure 1. The structure of quadrotor and its frames

II. DYNAMICS

A. Generalized Coordinates

We can set the generalized coordinate of the quadrotor with the position vector in earth frame $\boldsymbol{\xi} = (x, y, z)^T$ and the orientation of quadrotor referred to roll, pitch, and yaw vector $\boldsymbol{\zeta} = (\phi, \theta, \psi)^T$ respect to the earth frame as follows:

$$\boldsymbol{q} = (x, y, z, \phi, \theta, \psi)^T = (\boldsymbol{\xi}, \boldsymbol{\zeta})^T \tag{1}$$

The angular velocity vector $\boldsymbol{w}$=$(p, q, r)^T$, that represents the angular velocities around the body frame. It's related to the differential of the roll, pitch, and yaw by Fossen(2002)[8]

$$\boldsymbol{w} = W_\zeta \dot{\boldsymbol{\zeta}} \tag{2}$$

where

$$W_\zeta = \begin{pmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & S_\phi C_\theta \\ 0 & -S_\phi & C_\phi C_\theta \end{pmatrix} \tag{3}$$

where $S_{(\cdot)}$ and $C_{(\cdot)}$ represent sin$(\cdot)$ and cos$(\cdot)$. Then, the Lagrangian can be defined with translational kinetic energy, rotational kinetic energy potential energy by Raffo(2010).[9]

$$L(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2} m \dot{\boldsymbol{\xi}}^T \dot{\boldsymbol{\xi}} + \frac{1}{2} \dot{\boldsymbol{\zeta}}^T J \dot{\boldsymbol{\zeta}} - mgz \tag{4}$$

J is the inertia matrix of quadrotor in terms of the generalized coordinates $\boldsymbol{\zeta}$. The inertia matrix in the body frame is defined below:

$$I = \begin{pmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{pmatrix} \tag{5}$$

So the inertia matrix matrix in terms of the generalized coordinates $\boldsymbol{\zeta}$ can expressed with pre-defined transform matrix W_ζ as below:

$$J = W_\zeta^T I W_\zeta \tag{6}$$

B. Euler-Lagrange Equation

Force and Torque can be evaluate with Lagrangian by Euler-Lagrange equations as follows:

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{\boldsymbol{q}}} - \frac{\partial L}{\partial \boldsymbol{q}} = \boldsymbol{F} = (F_{\boldsymbol{\xi}}, \boldsymbol{\tau})^T \tag{7}$$

The translational force $F_{\boldsymbol{\xi}}$ can be written as follows:

$$\boldsymbol{F_\xi} = R_{BE}F_B = R_{BE}(0,0,u)^T \tag{8}$$

R_{BE} is a rotation transformation matrix from body frame to earth frame and u is the control input. R_{BE} is given below:

$$R_{BE} = \begin{pmatrix} C_\theta C_\psi & S_\theta S_\phi C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ C_\theta S_\psi & S_\theta S_\phi S_\psi + C_\phi S_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{pmatrix} \tag{9}$$

Control input u_f which represents the resultant vertical force in the body frame, can be expressed with lifting forces from the propellers. Lift forces f_i) can be determined with angular velocity ω_i as $f_i = k_i\omega_i^2$. The four rotors are same, so k_i is constant k.

$$u_f = \sum_{i=0}^{4} f_i \tag{10}$$

So the translational force equation can be expressed again as below:

$$m\ddot{\xi} + \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} = R_{BE}\begin{pmatrix} 0 \\ 0 \\ u_f \end{pmatrix} - \boldsymbol{k_t}\cdot\dot{\boldsymbol{\xi}} \tag{11}$$

$\boldsymbol{k}_t = (k_1, k_2, k_3)^T$ is the translational drag coefficient, and it is negligible at low speed. Similarly, the rotational torque $\boldsymbol{\tau}$ can be written as follows:

$$\begin{aligned} \boldsymbol{\tau} &= J\ddot{\boldsymbol{\zeta}} + (\dot{J}\dot{\boldsymbol{\zeta}} - \frac{1}{2}\frac{\partial}{\partial \boldsymbol{\zeta}}(\dot{\boldsymbol{\zeta}}^T J \dot{\boldsymbol{\zeta}}^T))\dot{\boldsymbol{\zeta}} \\ &= I\dot{\boldsymbol{w}} + \boldsymbol{w}\times(I\boldsymbol{w}) + k_r\boldsymbol{w} \end{aligned} \tag{12}$$

The torques in the body frame are expressed as below:

$$\boldsymbol{\tau} = (\tau_p, \tau_q, \tau_r)^T = \begin{pmatrix} l(f_4 - f_2) \\ l(f_3 - f_1) \\ d(f_1 - f_2 + f_3 - f_4) \end{pmatrix} \tag{13}$$

l is length between the center of the quadrotor and center of the blade. d is the ratio between the drag and thrust coefficient on the propeller. So we can reset the four control inputs from angular velocities of four blades as below:

$$\begin{aligned} u_\phi &= k(\omega_4^2 - \omega_2^2) = f_4 - f_2 \\ u_\theta &= k(\omega_3^2 - \omega_1^2) = f_3 - f_1 \\ u_\psi &= d(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \\ u_f &= k(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) = \Sigma_{i=1}^4 f_i \end{aligned} \tag{14}$$

C. Final Model

Then we can get the dynamic equations about $\boldsymbol{q} = (x, y, z, \phi,)$ as below:

$$\begin{aligned} \ddot{x} &= (u_f(c_\phi s_\theta c_\psi + s_\phi s_\psi) - k_1\dot{x})/m \\ \ddot{y} &= (u_f(c_\phi s_\theta s_\psi - s_\phi c_\psi) - k_2\dot{y})/m \\ \ddot{z} &= (u_f(c_\theta c_\phi) - mg - k_3\dot{z})/m \\ \ddot{\phi} &= (\dot{\theta}\dot{\psi}(I_y - I_z) - J\dot{\theta}\omega + lu_\phi)/I_x - k_4\dot{\phi} \\ \ddot{\theta} &= (\dot{\phi}\dot{\psi}(I_z - I_x) - J\dot{\phi}\omega + lu_\theta)/I_y - k_5\dot{\theta} \\ \ddot{\psi} &= (\dot{\phi}\dot{\theta}(I_x - I_y) + u_\psi)/I_z - k_6\dot{\psi} \end{aligned} \tag{15}$$

$$\boldsymbol{u} = \begin{pmatrix} u_\phi \\ u_\theta \\ u_\psi \\ u_f \end{pmatrix} = \begin{pmatrix} 0 & -k & 0 & k \\ -k & 0 & k & 0 \\ d & -d & d & -d \\ k & k & k & k \end{pmatrix}\begin{pmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{pmatrix} = U\begin{pmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{pmatrix} \tag{16}$$

where $\omega = \omega_2 + \omega_4 - \omega_1 - \omega_3$ is disturbance. Matrix U is nonsingular, so we can find the unique solution of the angular velocities of the blades ω_i^2 by choosing control inputs $u_\phi, u_\theta, u_\psi, u_f$.

If we choose the state vector as follows:

$$\boldsymbol{x} = (x, \dot{x}, y, \dot{y}, z, \dot{z}, \phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi})^T \tag{17}$$

Dynamic equations can be rewritten with state variables.

$$\begin{aligned} \dot{\boldsymbol{x}} &= \begin{pmatrix} x_2 \\ (u_f(C_{x_7}S_{x_9}C_{x_{11}} + S_{x_7}S_{x_{11}}) - k_1x_2)/m \\ x_4 \\ (u_f(C_{x_7}S_{x_9}S_{x_{11}} - S_{x_7}C_{x_{11}}) - k_2x_4)/m \\ x_6 \\ -g + (u_fC_{x_7}C_{x_9} - k_3x_6)/m \\ x_8 \\ (x_{12}x_{10}(I_y - I_z) - Jx_{10}\omega + lu_\phi)/I_x - k_4x_8 \\ x_{10} \\ (x_8x_{12}(I_z - I_x) - Jx_8\omega + lu_\theta)/I_y - k_5x_{10} \\ x_{12} \\ (x_8x_{10}(I_x - I_y) + u_\psi)/I_z - k_6x_{12} \end{pmatrix} \\ &= F(\boldsymbol{x}) + G(\boldsymbol{x})\boldsymbol{u} \end{aligned} \tag{18}$$

III. Control & Stability

We will apply three methods to control the quadrotor. The horizontal motion of the quadrotor is defined by the direction of the horizontal component of the thrust vector. So if the desired value of the horizontal position of the quadrotor is given, we can obtain the desired roll and pitch angle which make quadrotor to move desired position. So we will find the control input to stabilize the convergence of altitude, roll, pitch and yaw angle. If we obtain the control variables, we can get the outputs of actuators in blades as angular velocities ω_i from control inputs we achieved with any methods.

$$\begin{pmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{pmatrix} = U^{-1}\begin{pmatrix} u_\phi \\ u_\theta \\ u_\psi \\ u_f \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{2} & \frac{k}{4d} & \frac{1}{4} \\ -\frac{1}{2} & 0 & -\frac{k}{4d} & \frac{1}{4} \\ 0 & \frac{1}{2} & \frac{k}{4d} & \frac{1}{4} \\ \frac{1}{2} & 0 & -\frac{k}{4d} & \frac{1}{4} \end{pmatrix}\begin{pmatrix} u_\phi \\ u_\theta \\ u_\psi \\ u_f \end{pmatrix} \tag{19}$$

A. Feedback Linearization

We want to control the altitude, roll, pitch and yaw so let's consider x_5, x_7, x_9, x_11. We can see that only $\bar{\boldsymbol{x}} = (x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12})^T$ affect to roll, pitch, yaw and altitude. Then we can express with $\bar{\boldsymbol{x}}, \boldsymbol{u}$ as follows:

$$\begin{aligned}\begin{pmatrix}\ddot{x}_5\\ \ddot{x}_7\\ \ddot{x}_9\\ \ddot{x}_{11}\end{pmatrix} &= \begin{pmatrix}-g + (u_f C_{x_7} C_{x_9} - k_3 x_6)/m\\ (x_{12}x_{10}(I_y - I_z) - Jx_{10}\omega + lu_\phi)/I_x - k_4 x_8\\ (x_8 x_{12}(I_z - I_x) - Jx_8\omega + lu_\theta)/I_y - k_5 x_{10}\\ (x_8 x_{10}(I_x - I_y) + u_\psi)/I_z - k_6 x_{12}\end{pmatrix}\\ &= f(\bar{\boldsymbol{x}}) + g(\bar{\boldsymbol{x}})\boldsymbol{u}\end{aligned} \tag{20}$$

If the matrix $g(\bar{\boldsymbol{x}})$ is invertible, we can set input variables $\boldsymbol{u}$ as below:

$$\boldsymbol{u} = g(\bar{\boldsymbol{x}})^{-1}\left(\begin{pmatrix}\ddot{x}_{5d}\\ \ddot{x}_{7d}\\ \ddot{x}_{9d}\\ \ddot{x}_{11d}\end{pmatrix} - \alpha_1\begin{pmatrix}\dot{e}_5\\ \dot{e}_7\\ \dot{e}_9\\ \dot{e}_{11}\end{pmatrix} - \alpha_0\begin{pmatrix}e_5\\ e_7\\ e_9\\ e_{11}\end{pmatrix} - f(\bar{\boldsymbol{x}})\right) \tag{21}$$

where $e_i = x_i - x_{id}$. Then the eq(20) goes below:

$$\ddot{e}_i + \alpha_1\dot{e}_i + \alpha_0 e_i = 0 \tag{22}$$

where $i = 5, 7, 9, 11$. If we set $\alpha_0, \alpha_1 > 0$, The characteristic equation of it has the solution in left real plane. So the e_z goes zero when time goes infinite. We can also show it with the Lyapunov function. Let's set the Lyapunov candidate function as below:

$$V = \frac{1}{2}\alpha_0 e_i^2 + \frac{1}{2}\dot{e}_i^2 \tag{23}$$

Then check the differential of it.

$$\begin{aligned}\dot{V} &= \alpha_0 e_i\dot{e}_i + \dot{e}_i\ddot{e}_i\\ &= \alpha_0 e_i\dot{e}_i + \dot{e}_i(-\alpha_1\dot{e}_i - \alpha_0 e_i)\\ &= -\alpha_1\dot{e}_i^2\end{aligned} \tag{24}$$

The Lyapunov function is positive definite function(pdf), so it globally converges: $e_i \to 0, \dot{e}_i \to 0$. Now we can set the altitude as desired value.

B. Sliding Mode Control

Let's consider the dynamic equation of the roll angle.

$$\ddot{\phi} = (\dot{\theta}\dot{\psi}(I_y - I_z) - J\dot{\theta}\omega + lu_\phi)/I_x - k_4\dot{\phi} + g_1 \tag{25}$$

Where $d_1 = \frac{J}{I_x}\dot{\theta}w$ is associated by disturbance which is unknown, but it is assumed to be bounded by $\hat{d_1}$, and g_1 is the term of unknown dynamic but assumed to be bounded to $\hat{g_1}$. In nominal case, we can consider as follows::

$$\ddot{\phi} = \dot{\theta}\dot{\psi}\frac{I_y - I_z}{I_x} - \frac{J}{I_x}\dot{\theta}\omega + \frac{lu_\phi}{I_x} - k_4\dot{\phi} \tag{26}$$

To use the sliding mode control, we can set the sliding surface s as below:

$$s_\phi = (\frac{d}{dt} + \lambda_\phi)z_\phi = \dot{z}_\phi + \lambda_\phi z_\phi \tag{27}$$

where $z_\phi = \phi - \phi_d$ is the error of the roll angle. Let's check the differential of the sliding surface.

$$\begin{aligned}\dot{s}_\phi &= \ddot{\phi} - \ddot{\phi}_d + \lambda_\phi\dot{z}_\phi\\ &= \dot{\theta}\dot{\psi}\frac{I_y - I_z}{I_x} - \frac{J}{I_x}\dot{\theta}\omega + \frac{lu_\phi}{I_x} - k_4\dot{\phi} - \ddot{\phi}_d + \lambda_\phi\dot{z}_\phi\end{aligned} \tag{28}$$

So we need to make $s_\phi\dot{s}_\phi < -\eta|s_\phi|$, set the control input u_ϕ as follows:

$$\begin{aligned}u_\phi &= \frac{I_x}{l}(-\dot{\theta}\dot{\psi}\frac{I_y - I_z}{I_x} + k_4\dot{\phi} + \ddot{\phi}_d - \lambda_\phi\dot{z}_\phi - K_1 sign(s_\phi)\\ &= \frac{I_x}{l}(-x_{10}x_{12}\frac{I_y - I_z}{I_x} + k_4 x_8 + \ddot{x}_{7d}\\ &\quad - \lambda_\phi(\dot{x}_{7d} - x_8) - K_1 sign(s_\phi)\end{aligned} \tag{29}$$

Then we can analyze $s_\phi\dot{s}_\phi$.

$$\begin{aligned}s_\phi\dot{s}_\phi &= s_\phi(d_1 + g_1 - K_1 sign(s_\phi))\\ &\le (\hat{d_1} + \hat{g_1} - K_1)|s_\phi|\end{aligned} \tag{30}$$

If we set $K_1 = \hat{d_1} + \hat{g_1} + \eta$, then the sliding condition is satisfied. Which means that if we choose the Lyapunov candidate function as follows,

$$V = \frac{1}{2}s_\phi^2 \tag{31}$$

we can check its time derivative is negative definite.

$$\dot{V} = s_\phi\dot{s}_\phi = -\eta|s_\phi| \tag{32}$$

So by setting control input u_ϕ as before, ϕ will converges and stay to the manifold $s_\phi = 0$ in finite time. To avoid from the chattering effect due to the discontinuous function $sign(\cdot)$, we can replace the $sign(\cdot)$ to the saturation function $sat(\cdot)$ defined as follows:

$$sat(s) = \begin{cases} sign(s) & \text{if } |s| \ge \rho\\ \frac{s}{\rho} & \text{if } |s| < \rho\end{cases} \tag{33}$$

where ρ is a boundary layer of the sliding surface.

By using same steps we used in roll, pitch and altitude control, we can show the controllable with sliding mode control by setting control inputs as below:

$$\begin{aligned}u_\theta &= \frac{I_y}{l}(-x_8 x_{12}\frac{I_z - I_x}{I_y} + k_5 x_{10} + \ddot{x}_{9d}\\ &\qquad - \lambda_\theta(\dot{x}_{9d} - x_{10})) - K_2 sat(s_\theta)\\ u_\psi &= I_z(-x_8 x_{10}\frac{I_x - I_y}{I_z} + k_6 x_{12} + \ddot{x}_{11d}\\ &\qquad - \lambda_\psi(\dot{x}_{11d} - x_{12})) - K_3 sat(s_\psi)\\ u_f &= \frac{m}{C_{x_7}C_{x_9}}(g + k_3 x_6 + \ddot{x}_{5d} - \lambda_z(\dot{x_{5d}} - x_6)) - K_4 sat(s_z)\end{aligned} \tag{34}$$

where

$$
\begin{aligned}
z_\theta &= x_{9d} - x_9 \\
z_\psi &= x_{11d} - x_{11} \\
z_z &= x_{5d} - x_5 \\
s_\theta &= \dot{z}_\theta + \lambda_\theta z_\theta \\
s_\psi &= \dot{z}_\psi + \lambda_\psi z_\psi \\
s_z &= \dot{z}_z + \lambda_z z_z \\
K_2 &= \hat{d_2} + \hat{g_2} + \eta \\
K_3 &= \hat{d_3} + \eta \\
K_4 &= \hat{g_4} + \eta
\end{aligned}
\tag{35}
$$

C. Backstepping Control

We know that we have to consider the two derivative of altitude, roll, pitch and yaw to make the control input term. So we can express the equations as follows:

$$
\begin{aligned}
\dot{x} &= f(x) + g(x)\nu \\
\dot{\nu} &= u
\end{aligned}
\tag{36}
$$

where the x can be a x_5, x_7, x_9, x_{11}. Let's define the virtual variables as below:

$$
\begin{aligned}
x_1 &= x \\
x_2 &= \dot{x}_1 = \dot{x} \\
z_1 &= x - x_d \\
z_2 &= \alpha(z_1) - x_2
\end{aligned}
\tag{37}
$$

where $\alpha(z_1) = \dot{x}_d + \beta_1 z_1$ $(\beta_1 > 0)$. Let's consider the Lyapunov candidate function:

$$
V = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \tag{38}
$$

Then check the derivative of it.

$$
\begin{aligned}
\dot{V} &= z_1\dot{z}_1 + z_2\dot{z}_2 \\
&= z_1(-z_2 - \beta_1 z_1) + z_2(\ddot{x}_d - \beta_1\dot{z}_1 - \dot{x}_2) \\
&= -\beta_1 z_1^2 + z_2(\ddot{x}_d + (\beta_1^2 - 1)z_1 + \beta_1 z_2 - \dot{x}_2)
\end{aligned}
\tag{39}
$$

where $\beta_2 > 0$. To obtain $\dot{V} < 0$, set the virtual variable as below:

$$
\begin{aligned}
\dot{x}_2 &= \ddot{x}_d + (\beta_1^2 - 1)z_1 + (\beta_1 + \beta_2)z_2 \\
z_1 &= x_1 - x_d \\
z_2 &= \dot{x}_d - k_1 z_1 - x_2
\end{aligned}
\tag{40}
$$

So we can get the input variables by applying x to x_5, x_7, x_9, x_{11}.

$$
\begin{aligned}
u_\phi &= \frac{I_x}{l}(-x_{10}x_{12}\frac{I_y - I_z}{I_x} + k_4 x_8 + \ddot{x}_{7d} \\
&\qquad + z_1(\beta_1^2 - 1) + z_2(\beta_1 + \beta_2) \\
u_\theta &= \frac{I_y}{l}(-x_8 x_{12}\frac{I_z - I_x}{I_y} + k_5 x_{10} + \ddot{x}_{9d} \\
&\qquad + z_1(\beta_1^2 - 1) + z_2(\beta_1 + \beta_2) \\
u_\psi &= I_z(-x_8 x_{10}\frac{I_x - I_y}{I_z} + k_6 x_{12} + \ddot{x}_{11d} \\
&\qquad + z_1(\beta_1^2 - 1) + z_2(\beta_1 + \beta_2) \\
u_f &= \frac{m}{C_{x_7}C_{x_9}}(g + k_3 x_6 + \ddot{x}_{5d} + z_1(\beta_1^2 - 1) + z_2(\beta_1 + \beta_2))
\end{aligned}
\tag{41}
$$

IV. Fault Tolerant Control

We constructed the controller of the quadrotor under the fault-free condition. But if one of the four blade's actuators is in failure so not able to make upward lift force, we have to control the drone with remain three fully working actuators. We call it the Fault Tolerant Control(FTC). If we looses the one actuator to work, it means once control input will be dependent to the other three control inputs. So one of the altitude, roll, pitch and yaw variables have to loss the controllability. But the important variables are altitude, roll and pitch, that might affect the stability or collision of the quadrotor with small change. So we can loss the control of yaw, and control the other remain three variables by loosing the heading of the quadrotor.
Let's consider of the failure on 2nd blade actuator. The second column of the eq(16) goes to zero, so only three input variables are independent.

$$
\begin{pmatrix} u_\phi \\ u_\theta \\ u_\psi \\ u_f \end{pmatrix} = \begin{pmatrix} 0 & 0 & k \\ -k & k & 0 \\ d & d & -d \\ k & k & k \end{pmatrix} \begin{pmatrix} w_1^2 \\ w_3^2 \\ w_4^2 \end{pmatrix} \tag{42}
$$

So we can express $u_\psi = \frac{d}{k}(u_f - 2u_\phi)$. If the desired roll and pitch are chosen to zero and altitude to be a constant, $x_6, x_8, x_{10} \to 0$. So if the time goes enough to converge to desired values, the input variables goes below regardless to the three methods we used.

$$
\begin{aligned}
u_\phi &= \frac{I_x}{l}(-x_{10}x_{12}\frac{I_y - I_z}{I_x} + k_4 x_8) \to 0 \\
u_\theta &= \frac{I_y}{l}(-x_8 x_{12}\frac{I_z - I_x}{I_y} + J x_8 \omega / I_y + k_5 x_{10} \to 0 \\
u_f &= \frac{m}{C_{x_7}C_{x_9}}(g + \frac{k_3}{m}x_6) \to -mg
\end{aligned}
\tag{43}
$$

So the dependent input variable u_ψ goes below:

$$
u_\psi = \frac{d}{k}(u_f - 2u_\phi) \to -\frac{dmg}{k} \tag{44}
$$

The angular velocity in z_B-axis $r$can be expressed below:

$$
\dot{r} = \frac{k_6 r + u_\psi}{I_z} \to \frac{k_6 r - dmg/k}{I_z} \tag{45}
$$

So the angular velocity of yaw in body frame which we can't control retains boundedness:

$$
r \to \frac{dmg}{kk_6} \tag{46}
$$

It means when one blade goes in faulty, the altitude, roll and pitch can be controlled and converges to desired value. If roll and pitch goes zero and altitude be constant, the yaw direction spin goes constant angular velocity. So we can still use three controller constructed with three methods in fault-free condition by giving proper input variables according to the faulty condition.

V. Simulation Results

We chose one of the controller we constructed, feedback linearization method. Simulated the system to check the variables converges to desired trajectory. The simulation values and gains are used as below:

$$\begin{aligned} m &= 1.5kg \\ I_x &= I_y = 0.01kg \cdot m^2 \\ I_z &= 0.015kg \cdot m^2 \\ \Delta t &= 50ms \\ \alpha_1 &= 3 \\ \alpha_0 &= 3 \end{aligned} \tag{47}$$

First, we select the desired value to maintain fixed position.

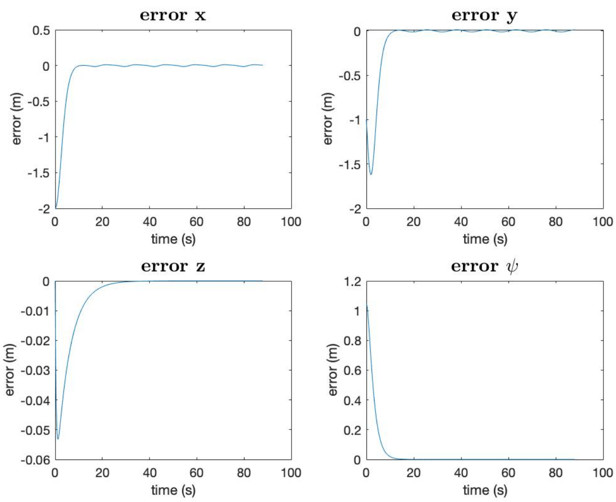

Figure 2. System errors with fixed desired position

We can check that errors converges to zero properly in finite time. Then we simulated for the cylindrical trajectory for given desired value.

$$\begin{aligned} x_d &= 1 + \cos\frac{t}{2} \\ y_d &= 1 + \sin\frac{t}{2} \\ z_d &= \frac{t}{6} \end{aligned} \tag{48}$$

We can see that the quadrotor is well controlled for time varying trajectory.

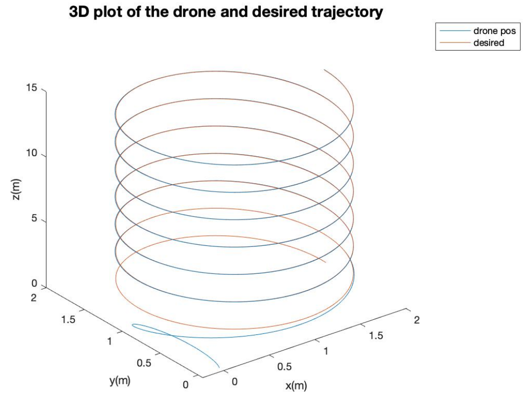

Figure 3. Quadrotor position and desired cylindrical trajectory

VI. Conclusion

Feedback linearization, sliding mode and backstepping control are well applied to the nonlinear system of the quadrotor to derive the input variables that converges altitude, roll, pitch and yaw angle. In the faulty mode, three variables altitude, roll and pitch can be controlled properly by loss of the yaw controllable. Even we abandoned the control of the yaw, if the desirable roll and pitch are zero and altitude goes constant, not only spin of the yaw doesn't diverge, but also converges to a certain value. It means we can also control the quadrotor's position. Due to the main controller structure is same, if an actuator fault occurs, the control of the quadrotor can be maintained by giving different inputs regardless of which controller it is. But the problem of these controllers are choosing the gain coefficients manually. So to apply in the real system, we need to find the proper coefficients in controller by experiments.

VII. Suggestion

We constructed three controller of quadrotor. But only feedback linearization controller in fault-free condition has been tested by simulation. If we Simulate other controllers in fault-free condition and even in faulty condition, we can compare the performance of the convergence. The range of the gain that controls the quadrotor properly would also be the important factor of controller. Furthermore, we can design the assembled controller, such as switching to different controller when the faulty occurs. Developing the state estimator to estimate fault detection will be also important.

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